



# **Nature of Jumps in VIX: Market Timing Hedge Funds**

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# CBOE Volatility Index (VIX)

- The CBOE VIX is a key measure of market expectations of near-term volatility conveyed by S&P 500 Index (SPX) option prices.
  - Market volatility
  - VIX is generally negatively correlated with the SPX and often signifies financial turmoil.
  - Investor sentiment or “investor fear gauge” (Whaley, 2000)
  - An indicator of investor appetite for risk (Dash and Moran, 2005)
- While the spot VIX is difficult to replicate as a practical matter, there are markets in VIX futures and options.

# CBOE Volatility Index (VIX) (continued..)

- *Volatility*, and VIX in particular, is widely thought to influence *hedge fund returns*.
  - Hedge funds often use *derivatives, short-selling, and leverage* to generate returns *during extreme states of the equity market*, and this can lead to hedge funds being exposed to *higher-moment risks of the equity market*.
- There is some evidence that the *change in VIX* is correlated with proxies for the *credit, liquidity, and correlation risks*.

# VIX Movements

- Being able to distinguish between Brownian motion, small jumps, and large jumps in sample path of VIX movements, and determining the relative magnitude of those components should be a welcome to investors, portfolio managers, and regulators.



HOUSE  
FOR SALE

# Methodology

- The study uses the methodology developed by Aït-Sahalia (2004), Aït-Sahalia and Jacod (2009*a,b*) and Aït-Sahalia and Jacod (2010*a,b*) to detect if there exists Brownian motion, or infinite activities (small jumps), or finite activities (big jumps) in the ultra-high frequency VIX data.
- For this to work, the VIX data need to have a lot of depth; that is, highly traded, down to the 15 seconds in this study.

# Realized Power Variations

The basic methodology is to construct realized power variations of VIX increments,  $B$  and  $U$ , suitably truncated and sampled at different frequencies.

$$\Delta_n^i \ln VIX \equiv \ln VIX_{i\Delta_n}^* + \varepsilon_i - (\ln VIX_{(i-1)\Delta_n}^* + \varepsilon_{i-1})$$

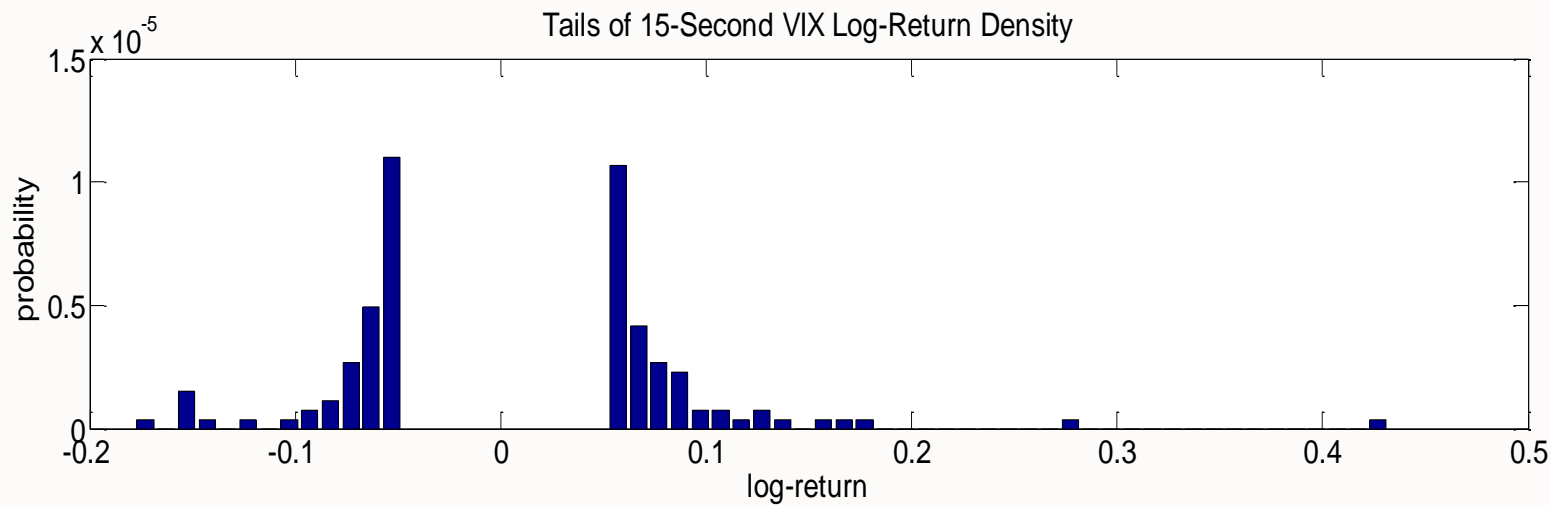
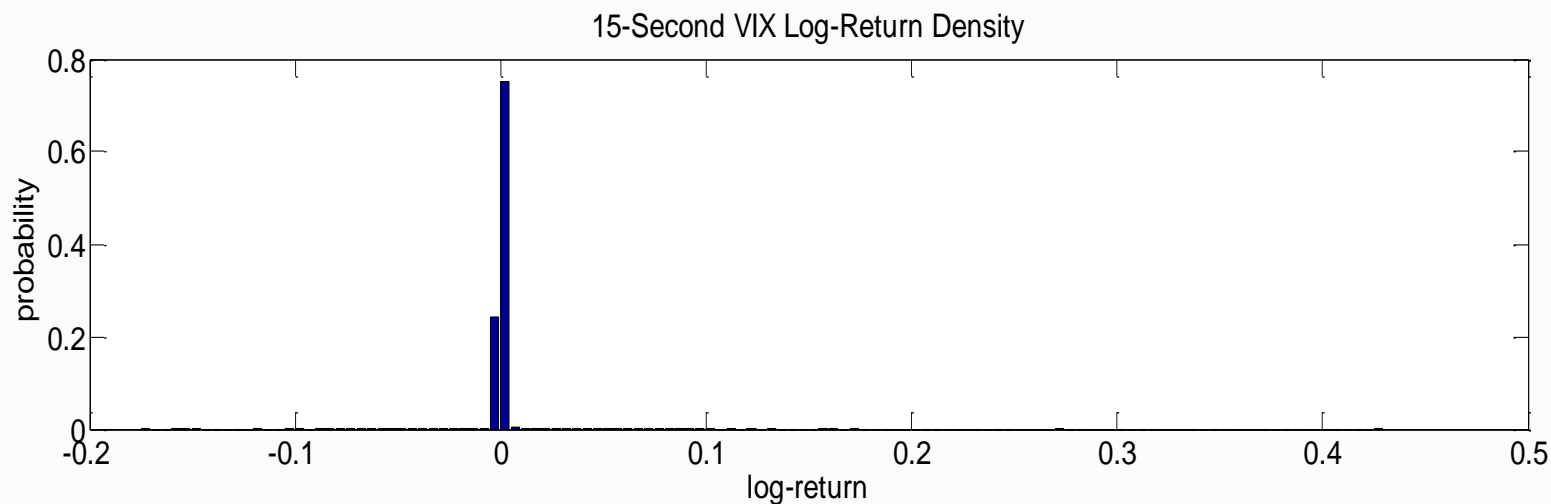
$$B(p, u_n, \Delta_n)_T = \sum_{i=1}^{\lceil T/\Delta_n \rceil} |\Delta_n^i \ln VIX|^p \times 1_{\{|\Delta_n^i \ln VIX| \leq u_n\}}$$

Retaining only the increments larger than  $u_n$  is written as:

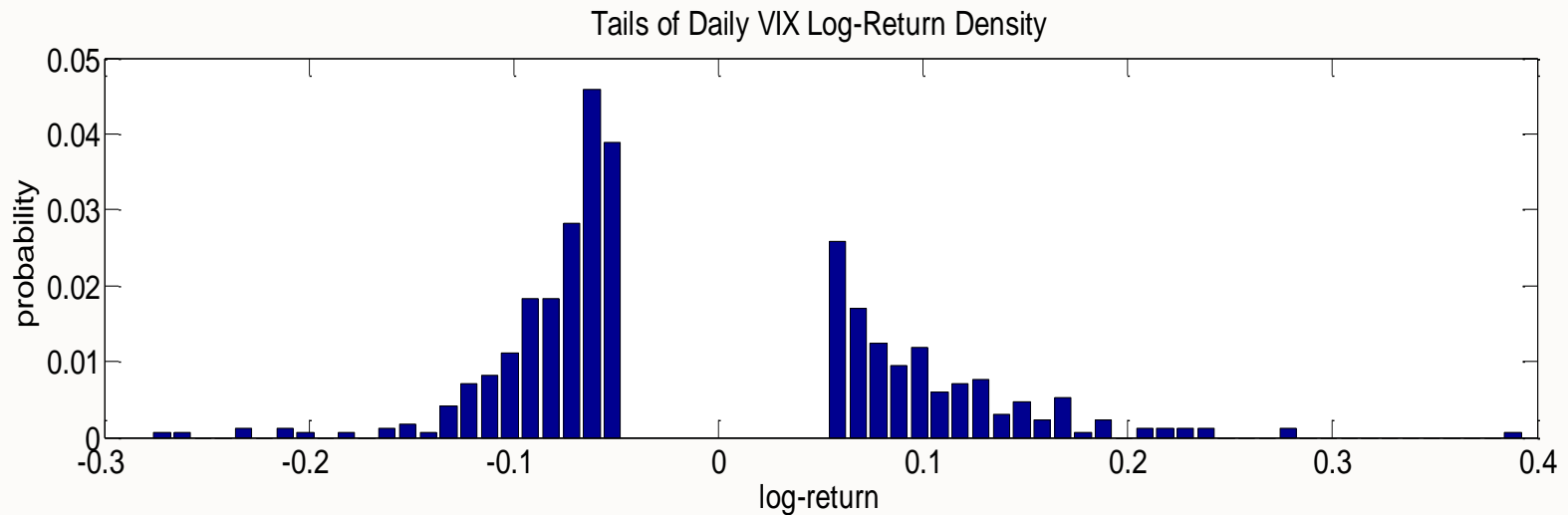
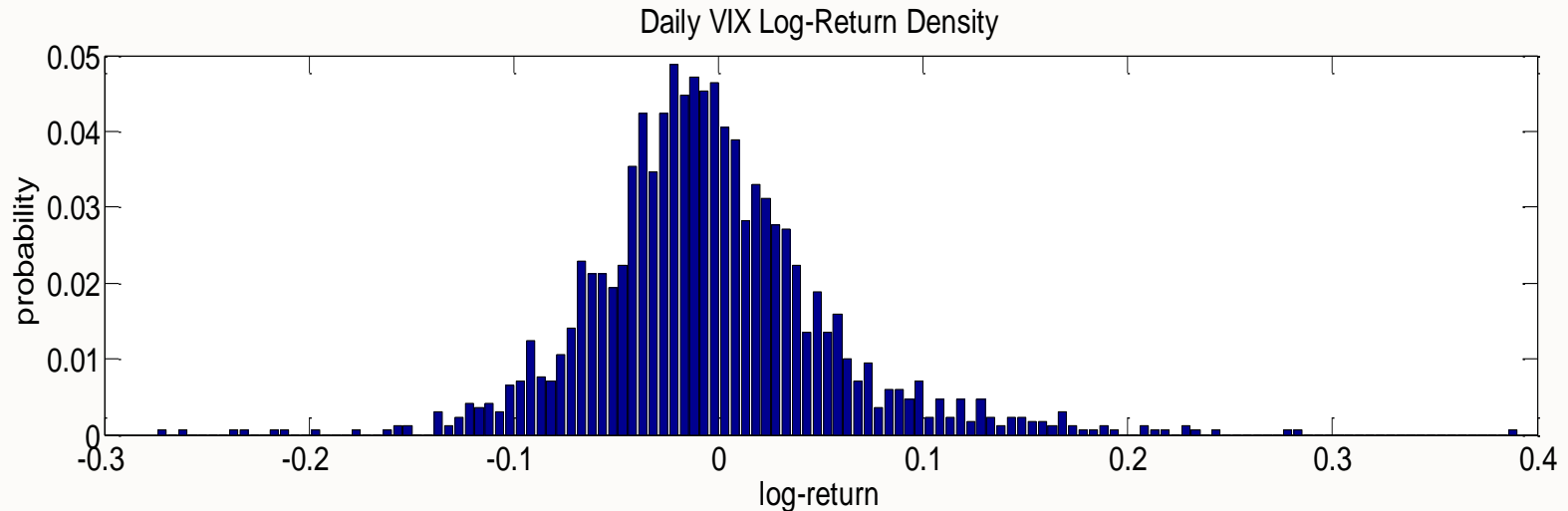
$$U(p, u_n, \Delta_n)_T = \sum_{i=1}^{\lceil T/\Delta_n \rceil} |\Delta_n^i \ln VIX|^p \times 1_{\{|\Delta_n^i \ln VIX| > u_n\}}$$

The study exploits the different asymptotic behavior of the variations  $B(p, u_n, \Delta_n)$  and  $U(p, u_n, \Delta_n)$  by varying the power  $p$ , the truncation level  $u_n$ , and the sampling frequency  $\Delta_n$ .

# Histogram of InVIX sampled at the 15 second frequency



# Histogram of InVIX sampled at the daily frequency



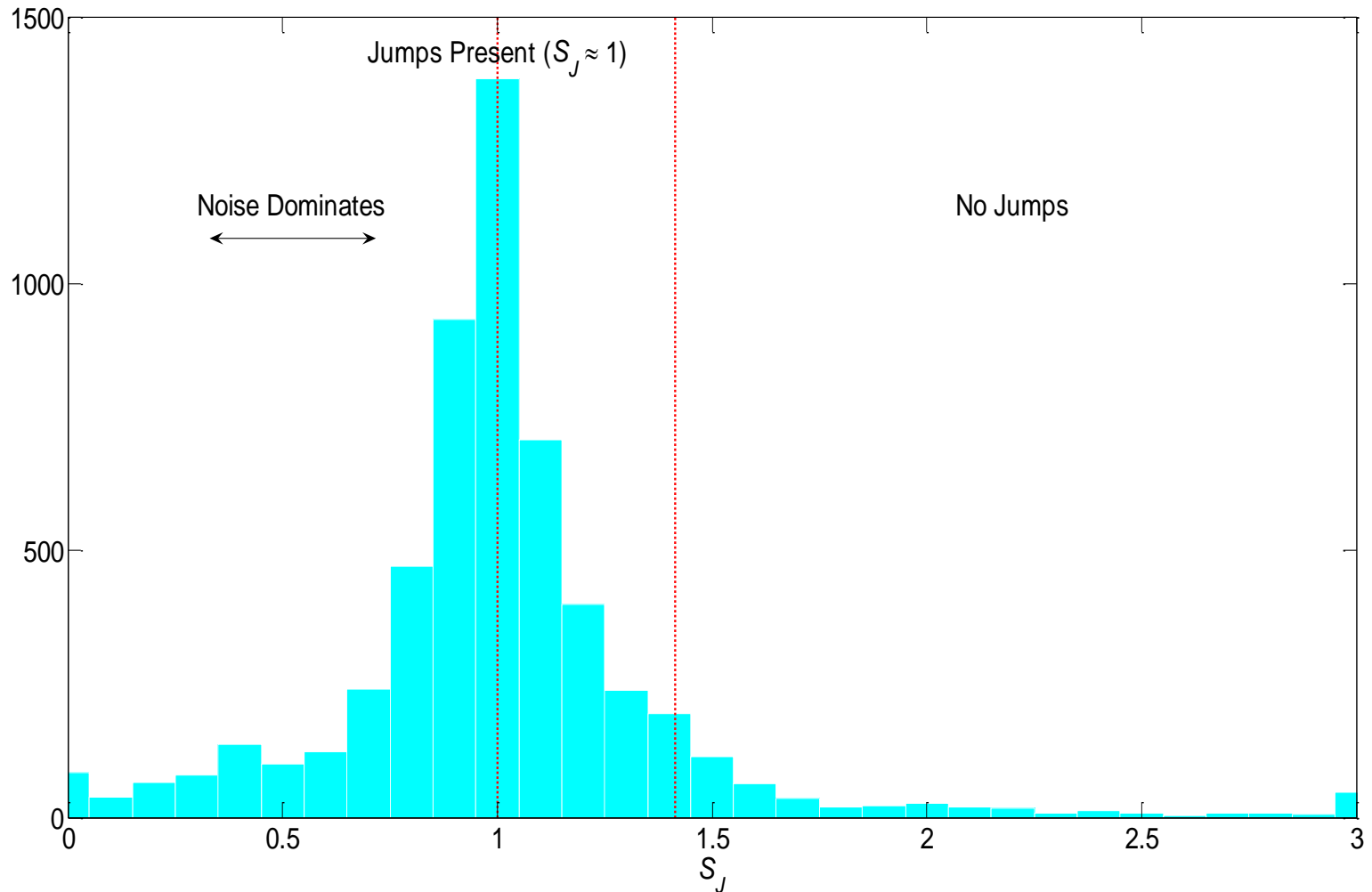


# Test statistic $S_J$ : the presence of jumps

$$S_J(p, k, \Delta_n) = \frac{B(p, \infty, k\Delta_n)}{B(p, \infty, \Delta_n)} \Big|_{p > 2, k \geq 2}$$

$\mathbb{P} \rightarrow \begin{cases} 1/k & \text{additive noise dominates} \\ 1/\sqrt{k} & \text{rounding error dominates (and jumps have finite activity)} \\ 1 & \text{jumps present and no significant noise} \\ k^{p/2-1} & \text{no jumps present and no significant noise} \end{cases}$

# Test statistic $S_J$ : the presence of jumps



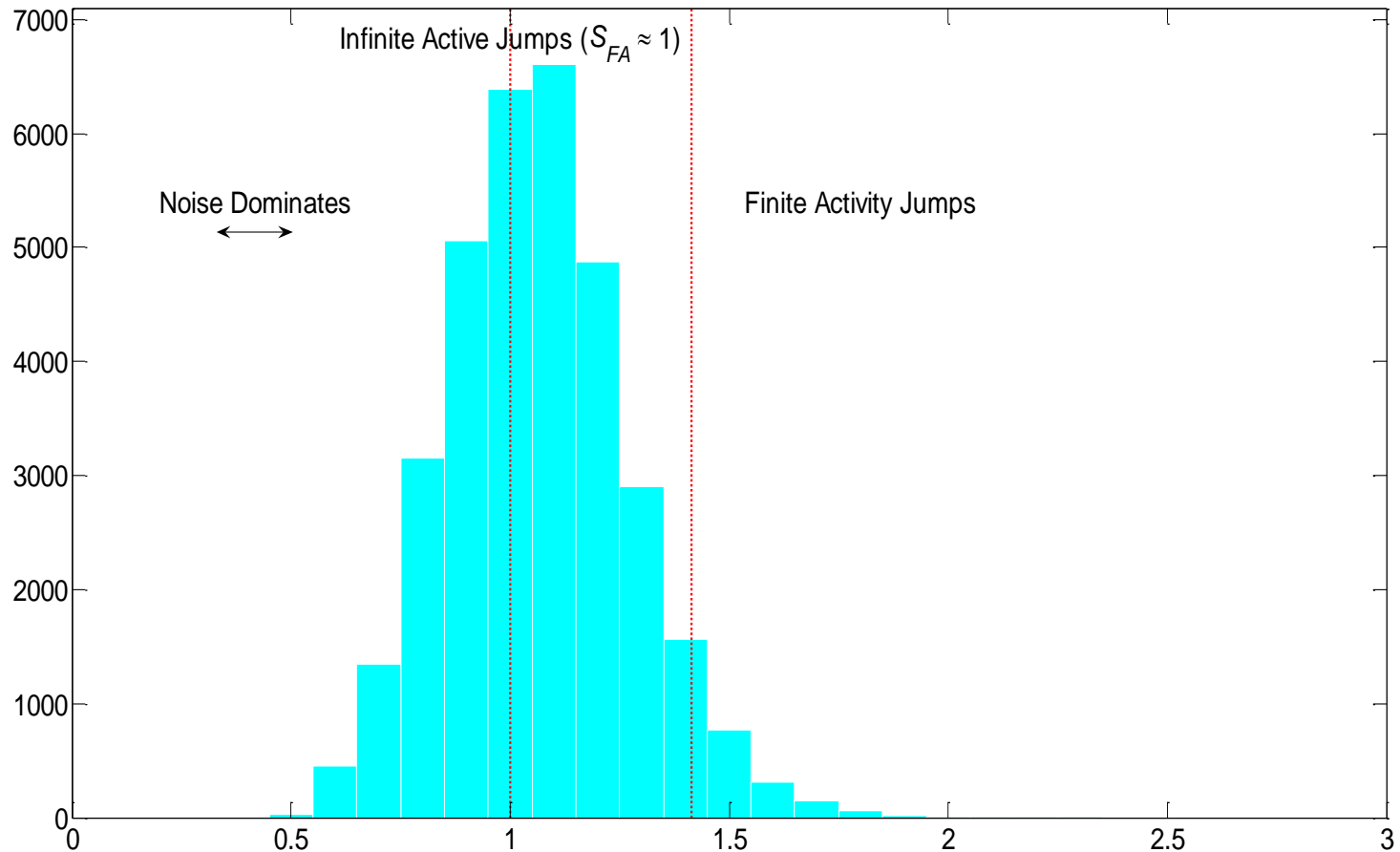
# Test statistic $S_{FA}$ : to test whether jumps have finite or infinite activity

$$S_{FA}(p, u_n, k, \Delta_n) =$$

$$\frac{B(p, u_n, k\Delta_n)}{B(p, u_n, \Delta_n)} \Big|_{p > 2, k \geq 2}$$

$$\xrightarrow{p} \begin{cases} 1/k & \text{additive noise dominates} \\ \text{no limit} & \text{rounding error dominates} \\ 1 & \text{infinite activity jumps present and no significant noise} \\ k^{p/2-1} & \text{finite activity jumps present and no significant noise} \end{cases}$$

# Test statistic $S_{FA}$ : to test whether jumps have finite or infinite activity

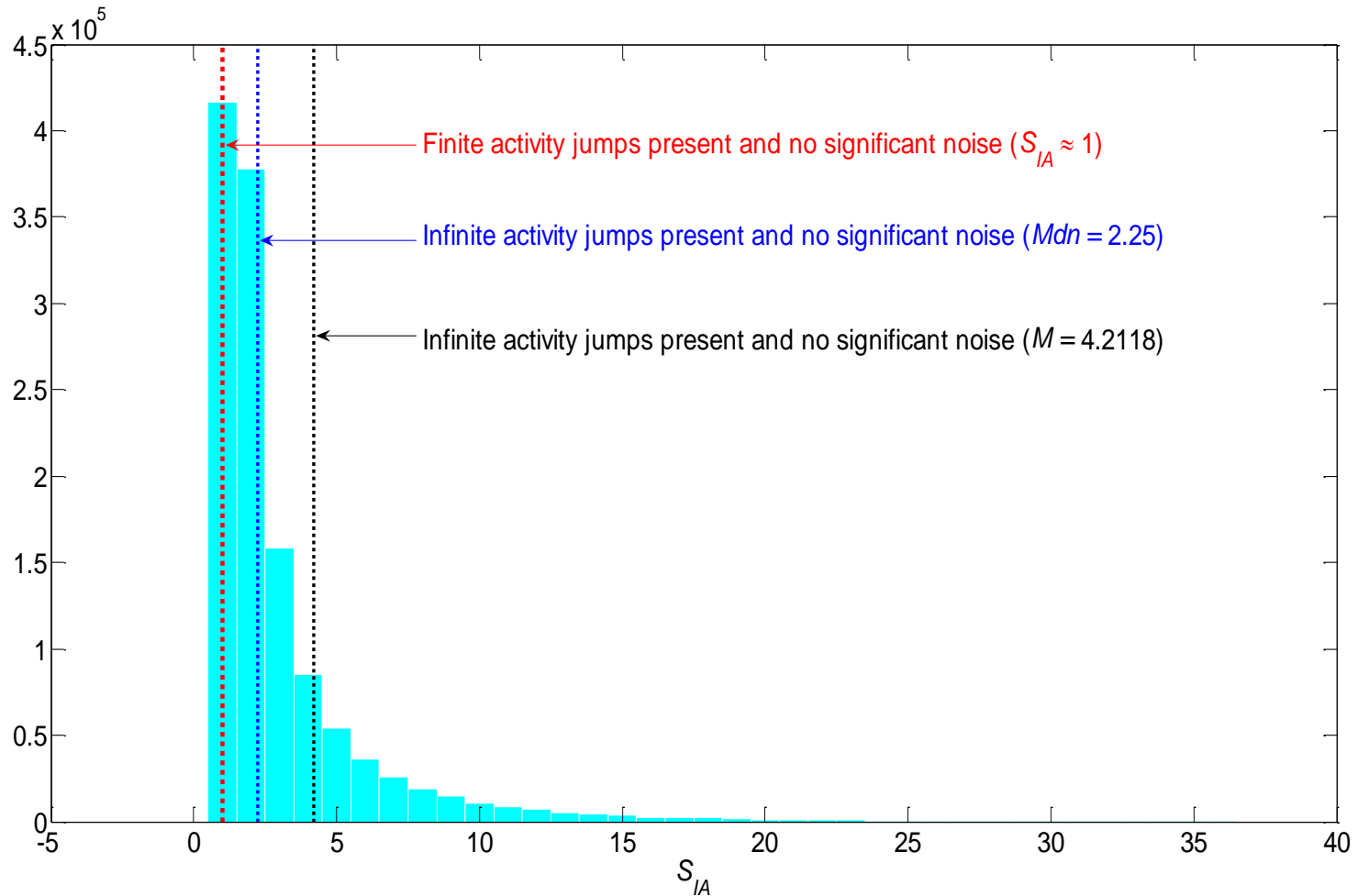


Test statistic  $S_{IA}$  : to test the null being  $\Omega_T^{i\beta}$  and the alternative of finite jump activity  $\Omega_T^f \cap \Omega_T^w$

$$S_{IA}(p, u_n, \gamma, \Delta_n) = \frac{B(p', \gamma u_n, \Delta_n) B(p, u_n, \Delta_n)}{B(p', u_n, \Delta_n) B(p, \gamma u_n, \Delta_n)} \Big|_{p' > p > 2, \gamma > 1}$$

$$\xrightarrow{\mathbb{P}} \begin{cases} 1 & \text{finite activity jumps present and no significant noise} \\ \gamma^{p'-p} & \text{infinite activity jumps present and no significant noise} \end{cases}$$

# Test statistic $S_{IA}$ : to test the null being $\Omega_T^{i\beta}$ and the alternative of finite jump activity $\Omega_T^f \cap \Omega_T^W$

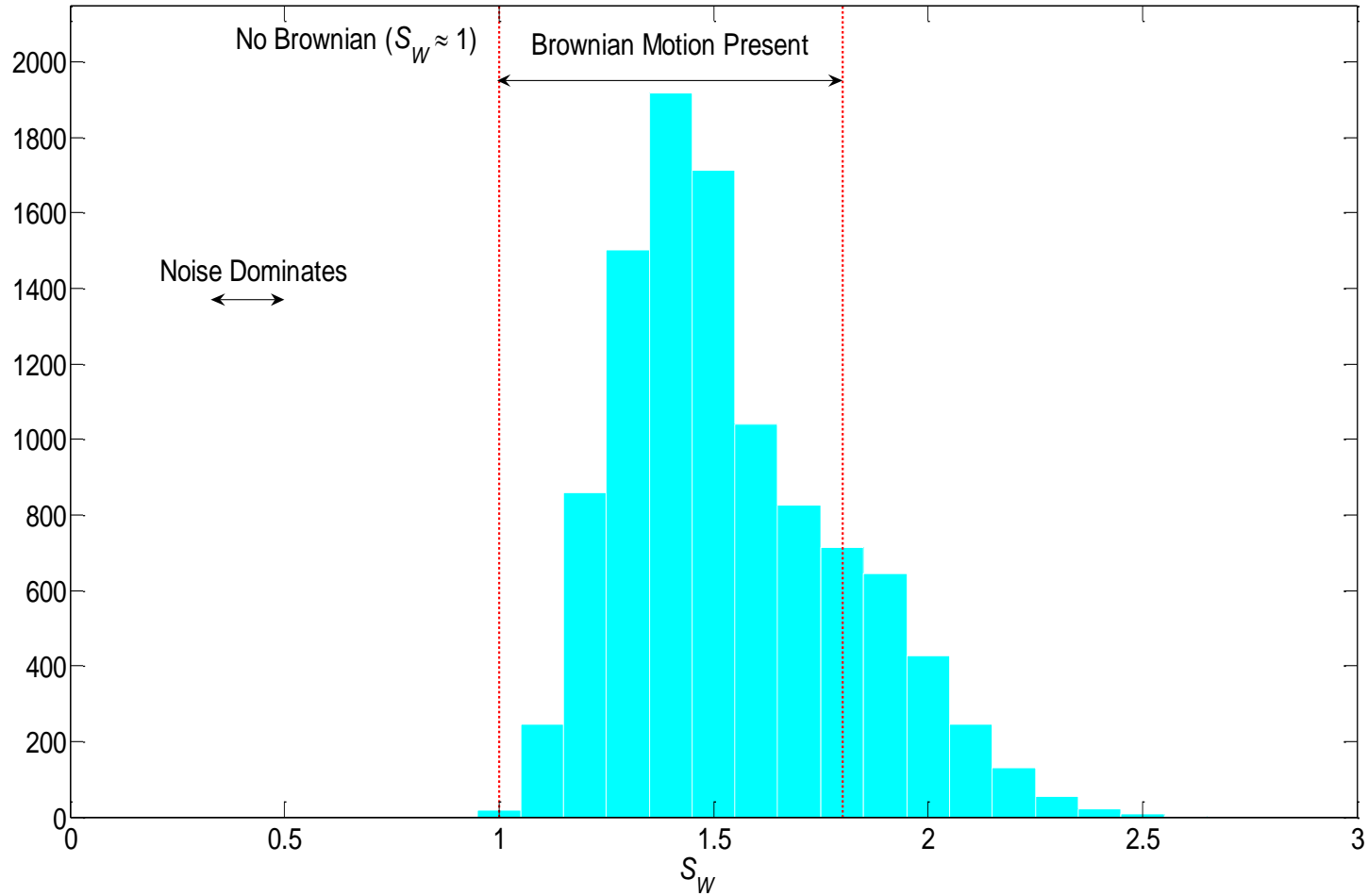


# Test statistic $S_W$ : to test whether Brownian motion is present

$$S_W(p, u_n, k, \Delta_n) = \frac{B(p, u_n, \Delta_n)}{B(p, u_n, k\Delta_n)} \Big|_{1 < p < 2, k \geq 2}$$

$$\xrightarrow{p} \begin{cases} 1/k & \text{additive noise dominates} \\ \text{no limit} & \text{rounding error dominates} \\ 1 & \text{No Brownian motion and no significant noise} \\ k^{1-p/2} & \text{Brownian motion present and no significant noise} \end{cases}$$

# Test statistic $S_W$ : to test whether Brownian motion is present





# The fraction of QV attributable to Brownian component

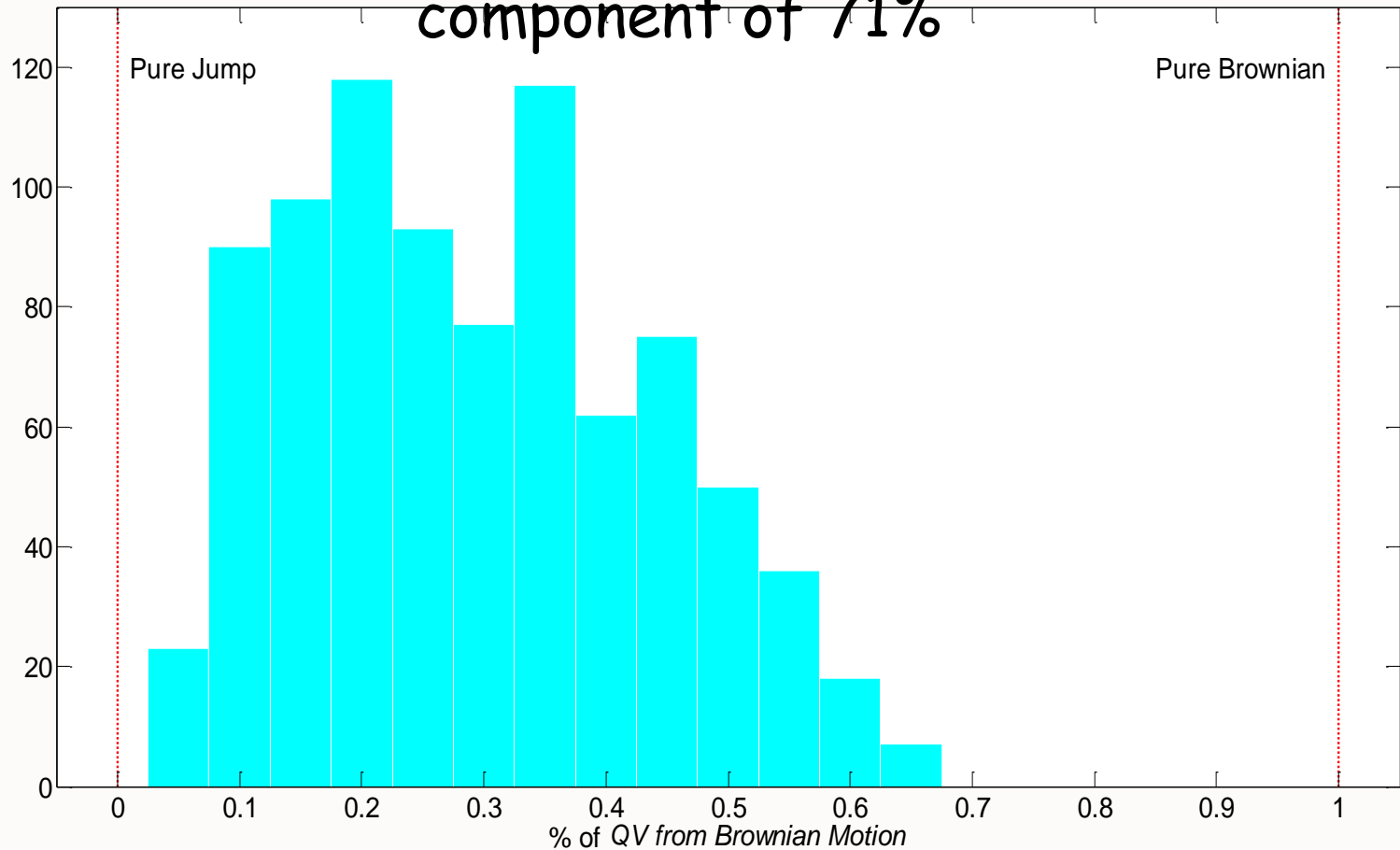
$$\left\{ \begin{array}{l} 0 \\ 0 \\ \frac{B(2, u_n, \Delta_n)}{B(2, \infty, \Delta_n)} \\ 1 - \frac{B(2, u_n, \Delta_n)}{B(2, \infty, \Delta_n)} \end{array} \right. \begin{array}{l} \text{additive noise dominates} \\ \text{rounding error dominates} \\ \text{\% of QV due to the continuous component} \\ \text{and no significant noise} \\ \text{\% of QV due to the jump component} \\ \text{and no significant noise} \end{array}$$

$$\text{\%QV due to big jumps} = \frac{U(2, \varepsilon, \Delta_n)}{B(2, \infty, \Delta_n)}$$

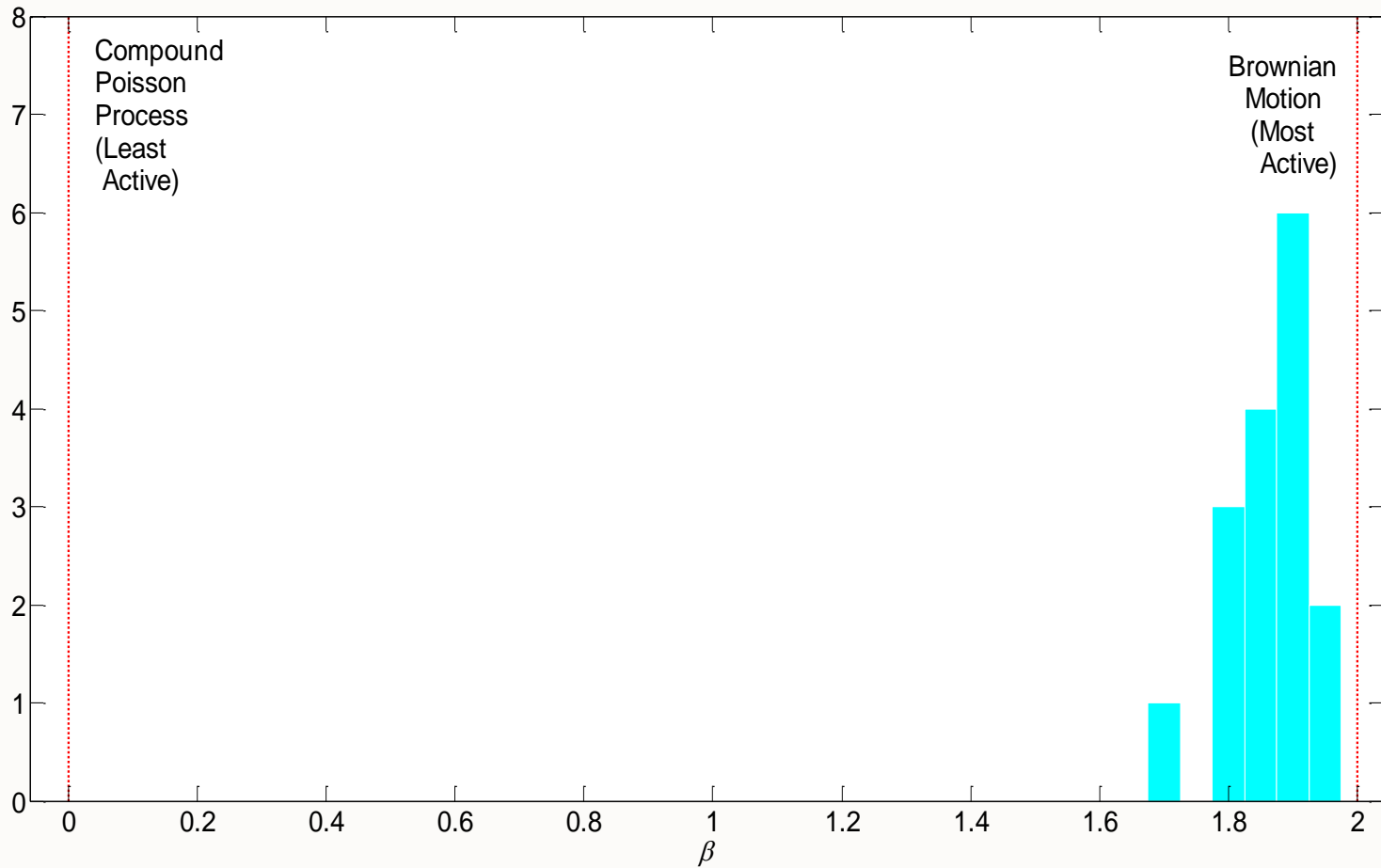
$$\text{\%QV due to small jumps} = 1 - \frac{B(2, u_n, \Delta_n)}{B(2, \infty, \Delta_n)} - \frac{U(2, \varepsilon, \Delta_n)}{B(2, \infty, \Delta_n)}$$

# The fraction of QV attributable to Brownian component

a continuous component of 29% and a jump component of 71%



# The degree of infinite jump activity $\beta$ in the range from 1.71 to 1.95



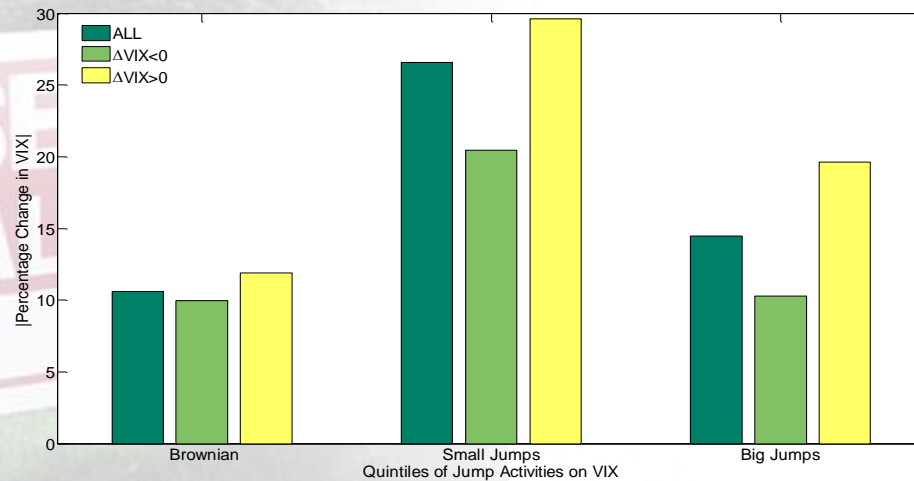
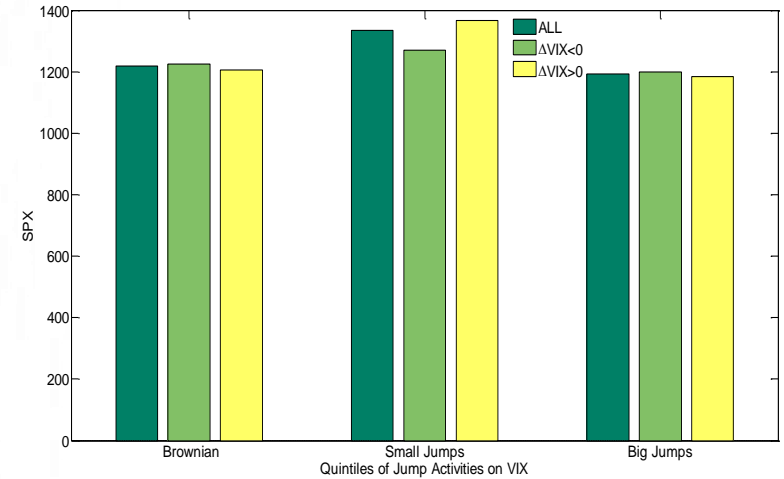
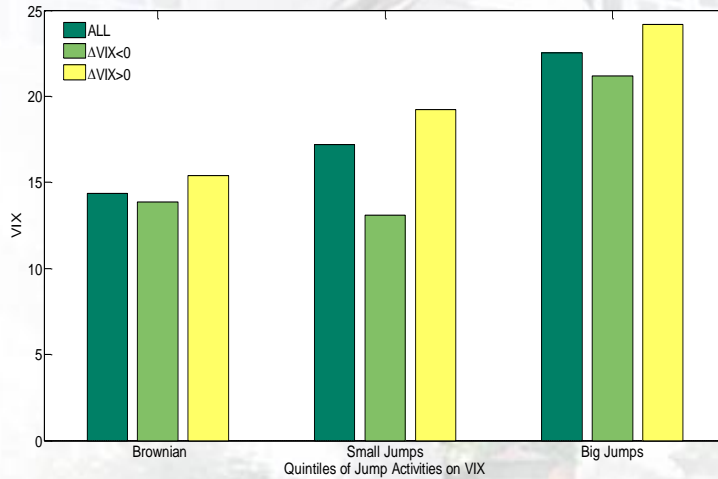
# Empirical Results

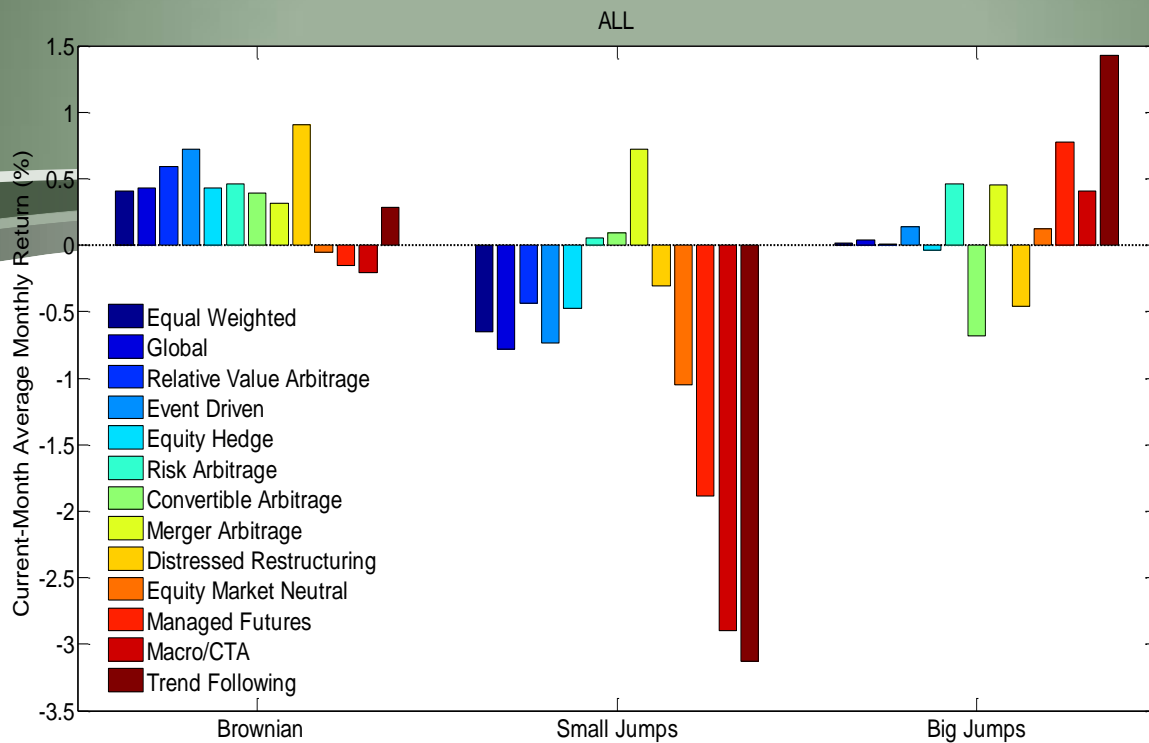
- Empirical results indicate that a continuous component, infinite and finite activity jumps are present in the ultra-high frequency VIX data.
- Relative to our findings, some (semi)parametric evidence for presence of jumps in the spot volatility has been proposed by Eraker, Johannes and Polson (2003), Eraker (2004), Broadie, Chernov and Johannes (2007), and Todorov (2010).

# Market Timing Hedge Funds

- Classify ultra-high VIX activities as Brownian motion, small jumps and large jumps, to see which component could predict hedge fund performance.
- November 2003 to July 2010
- 13 HFs:
  - (i) directional traders: Managed Futures, Macro/CTA and Trend Following,
  - (ii) relative value consisting of Relative Value Arbitrage and Convertible Arbitrage,
  - (iii) security selection consisting of Equity Hedge and Equity Market Neutral,
  - (iv) multi-process consisting of Event Driven, Merger Arbitrage, Distressed Restructuring and Risk Arbitrage,
  - (v) fund of funds consisting of Equal Weighted and Global.

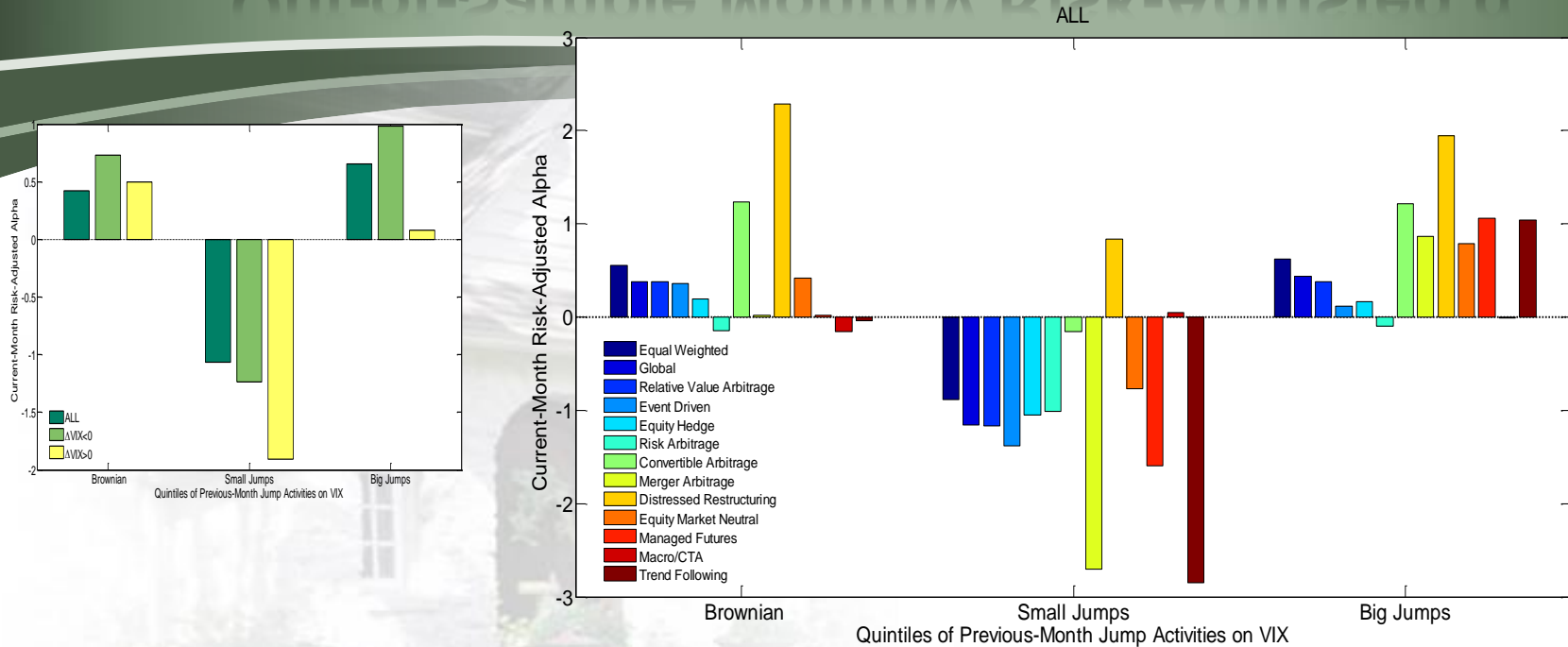
- The big jumps (60 months) : highest VIX, lowest SPX, medium  $|R_{VIX}|$
- The small jumps (3 months): medium VIX, highest SPX, highest  $|R_{VIX}|$
- Brownian motion (18 months) : smallest VIX, medium SPX, lowest  $|R_{VIX}|$





- *Trend Following, Managed Futures* and *Macro* outperform in the months following **big jumps** relative to the months following small jumps and Brownian motion.
- *Relative-value, equity-hedge* and *event-driven* funds to outperform in the months following *Brownian motion*.
- *Many hedge funds, in particular Arbitrage funds*, perform **best** in **calm markets**, and **worse** in the **jumping-around markets** that are driven by **small jumps on VIX**.
  - Arbitrage funds are in general *are short "liquidity"* as well as *short*

# Out-of-Sample Monthly Risk-Adjusted $\alpha$



- Conditioning on *large VIX jumps*, funds that *long volatility* deliver significantly higher out-of-sample  $\alpha$  relative to funds that *short volatility*, which coincides with extreme bear markets.
- On months that follow *Brownian motion* as a result of excluding the period with the 2008 financial crisis, hedge funds with *negative volatility exposure* tend to outperform those with positive volatility exposure.
- On the months that follow *small jumps* on VIX, possibly as a result of market illiquidity, the majority of hedge funds deliver negative returns, in particular when augmented with *positive change in VIX*.



# Conclusion

- Brownian, infinite and finite activity jumps are present in the ultra-high frequency VIX data, especially when taking into account the impact of market microstructure noise on various statistics.
- The total quadratic variation can be split into a continuous component of 29% and a jump component of 71%, which by construction is attributable to small and big jumps.
- Detecting jump activities on ultra-high frequency VIX data can help forecast cross-sectional differences in hedge fund performance through their exposure to long or short volatility risk evaluated conditional on different VIX jump quintiles.



**Thank You Very Much.**

# 林月能

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I joined the Department of Finance, National Chung Hsing University in 2005 as an Assistant Professor, was promoted to Associate Professor in 2008 and then to Professor of Finance in 2011. I obtained the PhD degree in Finance from Manchester Business School (MBS), United Kingdom. I have been a visitor at MBS, National University of Singapore, and Singapore Management University.

My research interest includes financial engineering, corporate finance, and volatility trading. I have published articles in several journals such as *Journal of Economic Dynamics and Control*, *Journal of Futures Markets*, *Review of Futures Markets*, *International Review of Economics and Finance*, *Applied Financial Economics*, *Review of Financial Economics*, *Annals of Economics and Finance*, and *Journal of Financial Studies*, and so on.